Minimizing DFA: Algorithm

① What if $M' = M$ ⇒ no optimization was necessary

② What if $|Q_{M'}| > |Q_M|$?

should not be possible!

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ALGORITHM: MinimizeDFAStates
   INPUT: DFA M = (Q, Σ, δ, q0, F)
   OUTPUT: M' s.t. minimum |Q| and equivalent behaviour as M
PROCEDURE:
   P := ∅ /* refined partition so far */
   T := { F, Q − F } /* last refined partition */
   while (P ≠ T):
      P := T
      T := ∅
      for(p ∈ P):
         find the maximal S ⊂ p s.t. splittable(p, S)
         if S ≠ ∅ then
            T := T ∪ {S, p − S}
         else
            T := T ∪ {p}
         end
```

accept states

non-accept states

as soon as $P = T$ ⇒ no further optimization can be done.

given a set P of states as a partition, can we find a proper subset of P that can be split from the rest of P.

splittable(p, S) holds iff there is $c \in \Sigma$ s.t.

1. $S \subset p$ (or equivalently: $p - S \neq \emptyset$)
2. Transitions via c lead all $s \in S$ to states in ***same partition*** $p1$ ($p1 \neq p$).

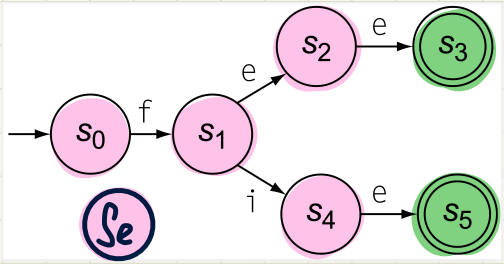
Partitions of States

e.g., $Q = \{s0, s1, s2, s3\}$

- Smallest number of partitions
- Largest number of partitions
- Partitions somewhere in-between
- Analogy from Software Testing: Equivalent Classes

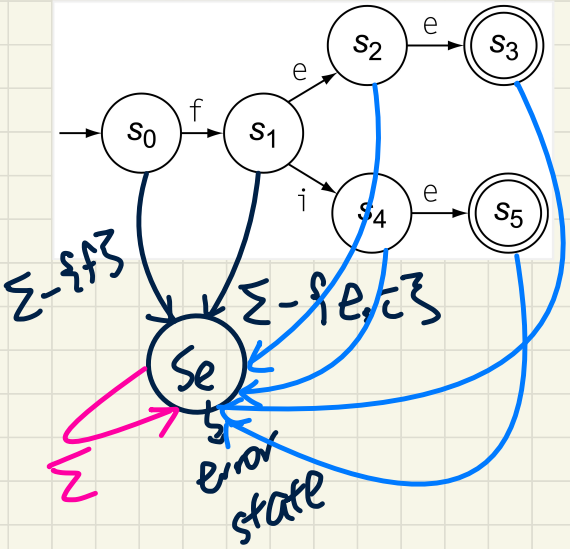
Minimizing DFA: Example (1)

(1)

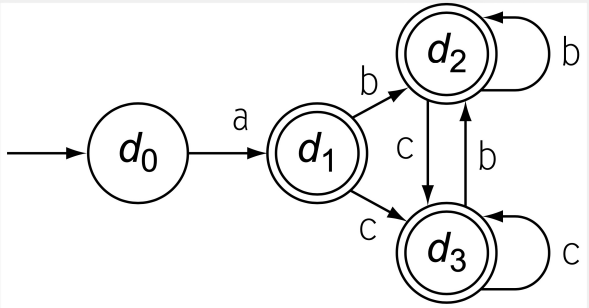


$P = \{ \{s_e, s_0, s_1, s_2, s_4\}, \{s_3, s_5\} \}$

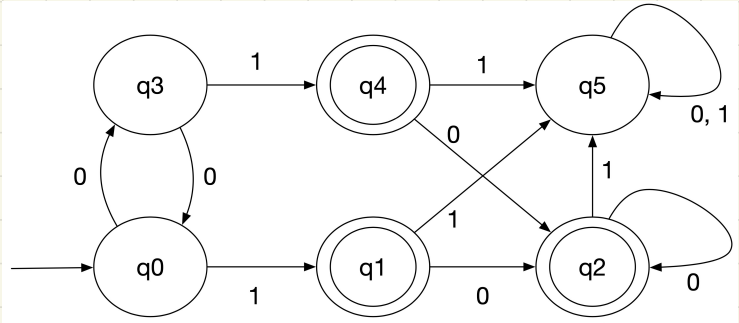
$\Sigma = \{a, b, \ldots, z\}$



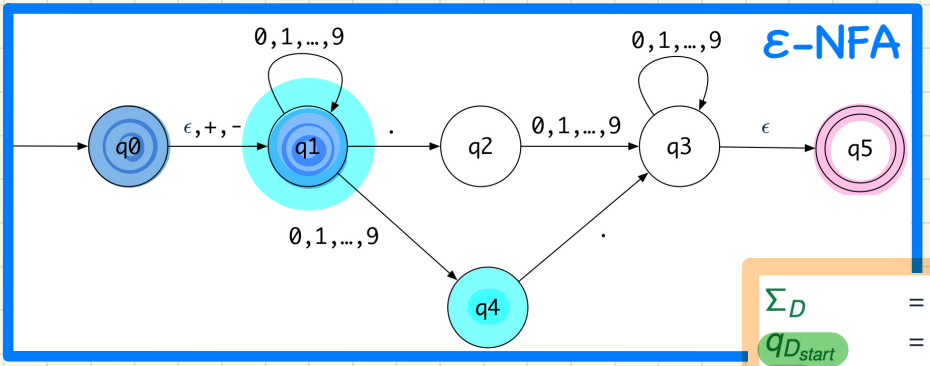
Minimizing DFA: Example (2)



Minimizing DFA: Example (3)



epsilon-NFA to DFA: Extended Subset Construction



$$
\begin{array}{lcl}
\Sigma_D & = & \Sigma_N \\
q_{D_{start}} & = & \mathtt{ECLOSE}(q_0) \\
F_D & = & \{\, S \mid S \subseteq Q_N \land S \cap F_N \neq \varnothing \,\} \\
Q_D & = & \{\, S \mid S \subseteq Q_N \land (\exists w \bullet w \in \Sigma^* \Rightarrow S = \hat{\delta}_N(q_0, w)) \,\} \\
\delta_D(S, a) & = & \bigcup \{\, \mathtt{ECLOSE}(s') \mid s \in S \land s' \in \delta_N(s, a) \,\}
\end{array}
$$

	$d \in 0..9$	$s \in \{+, -\}$	.
$\{q_0, q_1\}$	$\{q_1, q_4\}$	$\{q_1\}$	$\{q_2\}$
$\{q_1, q_4\}$	$\{q_1, q_4\}$	$\varnothing$	$\{q_2, q_3, q_5\}$
$\{q_1\}$	$\{q_1, q_4\}$	$\varnothing$	$\{q_2\}$
$\{q_2\}$	$\{q_3, q_5\}$	$\varnothing$	$\varnothing$
$\{q_2, q_3, q_5\}$	$\{q_3, q_5\}$	$\varnothing$	$\varnothing$
$\{q_3, q_5\}$	$\{q_3, q_5\}$	$\varnothing$	$\varnothing$

DFA